Vibration of Induction Machine Supplied with Voltage Containing Subharmonics and Interharmonics

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Abstract—In some power systems, voltage subharmonics and interharmonics of significant value may occur. They exert a harmful effect on various energy receivers, including induction motors. Previous works on an induction machine supplied with voltage containing subharmonics and interharmonics generally concern currents, power losses, rotational speed, electromagnetic torque and windings temperature. This paper reveals additional detrimental effect of voltage subharmonics and interharmonics: namely, excessive vibration of induction motors. It was found that voltage subharmonics of values a few times less than reported in real power systems may cause unacceptable levels of vibration.

Index Terms—induction motors, power quality, power system harmonics, harmonic distortion, vibrations, voltage fluctuations

I. INTRODUCTION

One of the most widespread electric motors is an induction cage machine. Due to common application of induction motors, their reliability and durability has a significant influence on various industrial processes. In some cases, failures of the machines may lead to high economic losses [1] and may even threaten human life and the natural environment. For example, in a vessel, damage of a prime mover driving fuel or a lubricating oil pump can cause stoppage of the propulsion engine and loss of ship manoeuvrability with significant consequences.

Faults of induction cage motors are of varied nature [1-4]. About 30-40% of failures are faults of stator windings [4], such as open circuit, coil-to-coil short circuit, and turn-to-turn short circuit, and are generally categorised as insulation faults. They typically result from causes including an improper impregnation process, carelessness during fabrication, overheating, and environmental agents such as moisture or chemicals [2,3]. Faults in a rotor, such as broken bars or end rings, occur much less often—they represent only about 5-10% of common faults of induction cage motors [4]. The most numerous type of fault is mechanical, which makes up about 50-70% of total breakdowns [4]. A weak point is the bearings—they make up about 40-50% of malfunctions [4]. An agent that dramatically reduces the life of bearings is vibration [5].

One of the causes of vibration of induction machines is a power quality disturbance such as voltage unbalance or voltage waveform distortions [6-10]. Voltage waveform distortions generally are associated with voltage harmonics. However, in a power network may appear voltage components of frequency not equal to integer multiples of the first voltage harmonic or even less than the fundamental frequency; they are called interharmonics and subharmonics (subsynchronous interharmonics), respectively. The power quality disturbances under consideration are caused by nonlinear loads, as inverters, cycloconverters, rolling mills, and arc furnaces [11-13]. Their source can also be photovoltaic plants and wind power stations [8,11,14-17], particularly under abnormal conditions. For example, [17] reports numerous subsynchronous resonance events in wind farms. During the analysed event, the voltage subharmonic level reached 1–2% $U_i$ (based on the additional information obtained from the authors of [17]). It is also worth mentioning that high levels of subharmonics and interharmonics are reported in the output voltage of some inverters [18,19]. It should be added that periodic voltage fluctuations can be considered as superposition of subharmonics and interharmonics [20]. Voltage fluctuations are caused by renewable sources of energy and high-power fluctuating loads such as rolling mills and railway tractions [11,15]. A similar effect may result from work of small receivers such as copy machines, X-rays equipments, lifts, refrigerators, and electric cookers [11].

Subharmonics are regarded as a particularly detrimental power quality disturbance. Among other things, they cause fluctuations of electromagnetic torque of generators, increase of magnetizing current of transformers, and light fluctuations and malfunctions of control systems [12,20]. In induction machines, they lead to oscillations of torque and rotational
speed, local saturation of magnetic circuits, and increase in magnetizing current and power losses, as well as overheating and thermal loss of life [12,19-31]. According to [22], a voltage subharmonic of order $h=0.1$ and value of 0.25% of the fundamental voltage component may cause a 17% reduction in the operational life of an exemplary 100 HP induction motor.

To protect energy receivers against malfunctions due to lowered voltage quality, relevant standards and power quality regulations have been developed. In many European countries, the national power quality rules are based upon EN 50160:2010 Voltage characteristics of electricity supplied by public distribution network [32]. Nevertheless, the standard [32] does not specify permitted levels of subharmonics and interharmonics; they “are under consideration, pending more experience”. Along with the notation in [32] are corresponding proposals to impose restrictive limitations on subharmonics [21,23]. The recommendations are justified primarily to avoid the damaging effect of subharmonics on heating and start-up of induction motors.

Research works on an induction machine supplied with voltage containing subharmonics and interharmonics [12,19-31] generally concern currents, power losses, rotational speed, electromagnetic torque and windings temperature. In [20], the effect of considered power quality disturbances on currents and speed fluctuations was analyzed. The impact of voltage subharmonics on flux, power losses and electromagnetic torque of an ultrahigh-speed induction machine was investigated in [19]. Power losses, windings temperature and electromagnetic torque under subharmonics, interharmonics and harmonics were considered in [23]. Induction motor thermal aging and loss of machine operational life due to subharmonics were presented in [12,21,22]. In [24], an innovative equivalent circuit, dedicated to induction machine under voltage fluctuation, was elaborated and results of research on current, power factor, power losses, rotational torque and speed fluctuations were presented. In [25,26], flux density, ohmic and core losses in the conditions of voltage fluctuations were investigated. In [27], the effect of subharmonics and interharmonics on currents and windings temperature was shown. Load-carrying capacity of induction machine supplied with voltage containing subharmonics was investigated in [28]. Flux density, currents and windings temperature under voltage subharmonics combined with voltage deviation were examined in [29]. Thermal transients of an induction machine under various power quality disturbances, including subharmonics, were analyzed in [30]. The influence of load properties on currents, power losses and windings temperature of an induction machine under subharmonics was analyzed in [31].

In summary, the effect of voltage subharmonics and interharmonics on vibration of induction cage machines has not been investigated yet. The main aim of this paper is to demonstrate that voltage subharmonics of values much less than those occurring in real power systems may cause excessive vibration, and, therefore, that power quality standards should be revised.

II. REASONS FOR THE VIBRATION OF INDUCTION MACHINES

The major internal sources of vibration of induction machines are radial and tangential electromagnetic forces, unbalanced centrifugal forces and various machine faults.

The radial electromagnetic forces involve mostly magnetostatic Maxwell forces (reluctance forces) and in a lesser degree, magnetostriuctive forces, which often are neglected [7,33]. Maxwell forces occur in the area of the air gap, while magnetostriiction forces are inside magnetic circuits of a rotor and a stator [7,33]. Both the forces produce vibration of the same frequency and deform magnetic sheet, especially teeth and try to reduce the distance between the rotor and the stator [7,33].

Radial electromagnetic forces occurring in the stator can be described as a force-wave expression [34,35]:

$$p(\alpha,t) = P(r,\omega) \cos(r \alpha - \omega t - \psi_p)$$

(1)

where $\alpha$ is angular coordinate, $t$ is time, $P(r,\omega)$ is force-wave amplitude, $r$ is force-wave order (mode), $\omega$ is force-wave angular frequency, $\psi_p$ is phase angle.

The radial electromagnetic forces are proportional to the squared flux density [7] and the force wave can be expressed as [34,35]:

$$p(\alpha,t) = \frac{B^2(\alpha,t)}{2\mu_0}$$

(2)

where $B(\alpha,t)$ is the flux density, $\mu_0$ is the permeability of free space.

Consequently, the fundamental current harmonic leads to vibration of its doubled frequency [7,33-35]. Additional frequency components caused by radial forces are generally interconnected with the presence of teeth. The magnetic field due to current flow in rotor bars causes forces appearing in stator teeth and, consequently, vibration [33]. The frequency of its main component, called the rotor bar passing frequency ($f_{rp}$), is given by (on the basis of [33] and [34]):

$$f_{rp} = f_1 \frac{z_r(1-s)}{p}$$

(3)

where $f_1$ is the frequency of the fundamental voltage component, $s$ is slip, $z_r$ is the number of rotor slots, $p$ is the number of pole pairs

The rotor bar passing frequency has sideband components resulting in high-frequency noise [33].

Under normal operating conditions, vibration due to electromagnetic forces should be insignificant, while under faulty conditions they can reach an excessive level [34,35]. A frequent cause of the breakdown of induction machines is
stator and rotor eccentricity [36]. This can be divided into three types: dynamic eccentricity, static eccentricity and mixed [36]. In the dynamic one, the axes of rotation does not coincide with the rotor axis [4,36] and centrifugal force is additionally superimposed on asymmetrical magnetic pull due to variable air gap thickness [37]. Further, in the static eccentricity both the axes coincide but the air gap thickness is nonuniform [3,36]. Consequently, asymmetrical electromagnetic force may result in dynamic eccentricity [3]. Vibration due to eccentricity [34,38] may contain among other things components corresponding to the rotational frequency, doubled rotational frequency and doubled line frequency with sidebands. A set of frequency components caused by various mechanical and electrical faults of induction machines is provided in [38].

High vibration level can be caused by tangential forces (Lorenz forces). Under machine internal faults or power quality disturbances, they may lead to significant torque pulsations, which results in stator and rotor torsional vibration [34,35]. Ref. [34,35] report extraordinary vibration of induction motors due to voltage unbalance and the presence of a DC component in the supply voltage.

The pulsating component of the electromagnetic torque under voltage fluctuations (which may be considered as a superposition of subharmonics and interharmonics [20]) can be calculated as (on the basis of [24]):

$$T_e = \frac{3p}{4} L_M \left[ -I_{sh}^2 I_{r0} + I_{sh} I_{r1}^* + I_{sh} I_{r1}^* - I_{sh}^* I_{r1}^* \right]$$ (4)

where $T_e$ is electromagnetic torque, $L_M$ is mutual inductance, $\omega$ indicates imaginary part of a complex quantity, subscripts $S$, $R$ concern a stator and a rotor, correspondingly, subscripts $I$, $sh$ and $ih$ refer to the fundamental, subharmonic and interharmonic components, respectively.

The values of fundamental, subharmonic and interharmonic components can be determined with a dedicated equivalent circuit, described with the following equations [24]:

$$U_{si} = \left( R_s + j\omega L_s + L_s \frac{d}{dt} \right) I_{si} + \left( j\omega L_{st} + L_M \frac{d}{dt} \right) I_{r1}$$ (5)

$$0 = \left( j\omega L_{st} + L_M \frac{d}{dt} \right) I_{si} + \left( R_s + j\omega L_s + L_s \frac{d}{dt} \right) I_{r1} - j\Omega_{00} (L_{si} I_{r0} + L_{r1} I_{r0})$$ (6)

$$0 = \left( j\omega L_{st} + L_M \frac{d}{dt} \right) I_{si} + \left( R_s + j\omega L_s + L_s \frac{d}{dt} \right) I_{r1} - j\Omega_{00} (L_{si} I_{r0} + L_{r1} I_{r0})$$ (7)

$$0 = \left( j\omega L_{st} + L_M \frac{d}{dt} \right) I_{si} + \left( R_s + j\omega L_s + L_s \frac{d}{dt} \right) I_{r1}$$ (8)

$$+ \left( j\omega L_{st} + L_M \frac{d}{dt} \right) I_{r1} - j\Omega_{00} (L_{si} I_{r0} + L_{r1} I_{r0})$$ (10)

where $\Omega_{00}$ is the average rotational speed, $\Omega_{00}$ is the amplitude of speed fluctuations, $\omega_0$ is the angular frequency of voltage fluctuation.

Exemplary results of investigations on torque pulsations are presented in [24].

To sum up, various faults and power quality disturbances may result in excessive vibration level.

III. PERMISSIBLE VIBRATION LEVEL

Recommendations concerning permissible vibration levels are provided, among the other things, in [39-42]. According to the standard IEC 60034-14 Rotating electrical machines - Part 14: Mechanical vibration of certain machines with shaft heights 56 mm and higher - Measurement, evaluation and limits of vibration severity [39], for small, rigid-mounted machines (with shaft height less than 132 mm), the vibration velocity should not exceed 1.3 mm/s. It is important to note that the standard does not apply for measurements in situ and machines coupled to any load or prime movers. More comprehensive is the standard ISO 20816-1 Mechanical vibration — Measurement and evaluation of machine vibration — Part 1: General guidelines [40]. Depending on vibration velocity, [40] distinguishes four zones, denoted as Zone A, Zone B, Zone C, and Zone D. Vibration level in Zone A generally corresponds to newly commissioned machines. Vibration within Zone B is considered long-term permissible, and within Zone C it is unacceptable for long-term operation. In practice, a machine can be operated only for a limited period of time. Finally, vibration within Zone D may cause damage to a machine. The standard [40] does not specify univocal boundaries between each zone. Appropriate indications are given in the anterior standard ISO 10816-1 Mechanical vibration -- Evaluation of machine vibration by measurements on non-rotating parts -- Part 1: General guidelines [41]. According to [41], for small electric motors
(with rated power up to 15 kW), vibration of velocity of at most 0.71 mm/s should be classified in Zone A, between 0.71 and 1.8 mm/s – in Zone B, 1.8 to 4.5 mm/s in Zone C, and above 4.5 mm/s in Zone D. It should be noted that the meaning of Zones A – D is the same in both standards [39,41].

IV. MEASUREMENT STAND

The measurement equipment consists of the induction machines under investigation, a supply system, power quality analysers, and a system for measuring vibration.

Two newly commissioned induction cage motors of 3SIE100L4B type, rated power 3 kW, and efficiency class IE3 are used in this study. A set of their parameters is presented in Table 1. Both machines are mounted on rigid frames (see Fig. 1), and one of them is additionally coupled with a DC generator supplying a resistor bank. For generation of subharmonic and interharmonics, a multi-machine system was applied (on the basis of [42]). It is composed of a transformer with experimentally chosen number of turns and two synchronous generators: Leroy Somer LSA 42.2M6 6/4 (S_{rat}= 23 kVA, I_{rat}= 33 A, x_{d}= 190\%, x_{q}= 100\%) and GTNSa132s2/08 (S_{rat}= 6.5 kVA, I_{rat}= 9.9 A). The former is driven with a rotational speed corresponding to the fundamental voltage harmonic and the latter to the subharmonic or interharmonic component. For reference tests (without injection of subharmonics and interharmonics) this alternator is disconnected. Voltage waveforms produced by both generators are summed by the transformer. Because of the limit of rotational speed, the maximal interharmonic frequency is 70 Hz. The content of subharmonics and interharmonics in the supply voltage and motor current was determined with a PC-based power quality analyser and a power quality estimator analyser [42], engineered in Gdynia Maritime University for commercial use and certified by the Polish Register of Shipping. For vibration measurement, a Bruel & Kjear system was employed, consisting of a 6-channel input module LAN-XI 51.2 kHz Type 3050, magnetically mounted accelerometers Type 4514 – B, an accelerometer calibrator Type 4294, and a recorder - PC computer with Bruel & Kjear software PULSE. As casings of the motors under investigations are made of aluminium, the accelerometers are fixed to an additional steel stud that is screwed into the motor (Fig. 1), in the vertical (V), horizontal (H), and longitudinal (L) directions. Additional measurement points are located on the DC generator and the motor frames in the V direction. The sampling rate was set to 4 kHz. The length of the test samples was set to approximately 20 sec. The signals were recorded after vibration steadying (approximately 5 s). It should be mentioned that, according to the authors’ multianual experience for calculation of the broad-band velocity (rms value) of vibration, the frequency range is assumed to be 10 – 500 Hz. The analyses were performed on the base of FFT signal transformation.

A simplified diagram of the measurement stand is presented in Fig. 2.

<table>
<thead>
<tr>
<th>TABLE 1</th>
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<tbody>
<tr>
<td>TYPE 3SIE100L4B INDUCTION CAGE MACHINE PARAMETERS</td>
</tr>
<tr>
<td>Rated power (kW)</td>
</tr>
<tr>
<td>Rated frequency (Hz)</td>
</tr>
<tr>
<td>Rated voltage (V)</td>
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<tr>
<td>Rated current (A)</td>
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<td>Rated power factor (-)</td>
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<tr>
<td>Rated efficiency (%)</td>
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<tr>
<td>Rated rotational speed (rpm)</td>
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<td>Weight (kg)</td>
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<td>Manufacturer</td>
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Fig. 1. Photograph of the investigated motors with measurement equipment. 1 – Induction motors, 2 – Accelerometers, 3 – DC generator, 4 – Power quality estimator analyser, 5 – Input module, 6 – PC computer with Bruel & Kjear software PULSE.

Fig. 2. A simplified diagram of the measurement stand.
V. RESULTS OF INVESTIGATIONS

A. Considered Cases

In this section are presented the results of investigations on vibration of an induction machine under positive-sequence subharmonics and interharmonics. All the depicted tests were conducted for the fundamental voltage component equal to its rated value for the following cases:

- Case A – Uncoupled motor,
- Case B – Coupled motor, driving the de-energised DC generator,
- Case C – Coupled motor, fully loaded with the DC generator.

Case A and Case B correspond to a machine in the idle condition: for example, in no-load period under continuous-operation duty S6 [44]. Additionally, the former could roughly model a machine driving a load of insignificant moment of inertia (in comparison to the motor moment) and the latter—of high inertia load. For each of the considered cases, an additional reference test was made for supply with practically sinusoidal voltage, without subharmonic or interharmonic injection. Exemplary spectra of the vibration velocity for reference tests and the accelerometer mounted in direction H are shown in Fig. 3. In the diagram a linear scale was applied, because our intention was readable presentation of the frequency components that have the most important contribution to the broad-band velocity of vibration. The logarithmic scale makes that the major components became less readable. As the velocity was computed on the basis of accelerometer indications, the spectra contain additional low-frequency components resulting from integration constant. The highest vibration components appear for frequencies 25 Hz (Case A, Case B) and 24.25 Hz (Case C), which approximately correspond to the motor rotational frequency (the rotational speed is 1499, 1497 and 1462 rpm for Cases A, B and C, respectively). The other significant spectra components are of frequency 75 Hz (Case B) and 100 Hz (Case C). The broad-band velocity is about 0.60, 1.40, and 0.80 mm/s for Cases A, B, and C, respectively. For comparison, in direction V, the analogous values are 0.43, 0.96, and 0.45 mm/s, and for the DC generator they are 0.30 and 0.48 mm/s for Cases B and C, respectively.

In summary, the vibration levels presented above under reference tests correspond to Zone A and Zone B, and for Case A they additionally fulfill the requirements of [39].

B. Vibration under Voltage Subharmonics and Interharmonics

Below, in Figs. 4, 5 and 6, the broad-band velocity of vibration vs frequency of subharmonic and interharmonic components \( f_{sh} \) and \( f_{ih} \), respectively) is presented for the considered cases. All the depicted tests were conducted for the fundamental voltage component equal to its rated value, injection of a subharmonic or interharmonic component of value \( U_{sh/ih} = 1\% U_{rat} \) and the following frequencies:

- Case A – 5, 10, …, 70 Hz (except 50 Hz) and additionally 26.5, 21, 22, …, 29 Hz
- Case B and Case C – 5, 10, …, 70 Hz (except 50 Hz)

It should be noted that the value \( U_{sh/ih} = 1\% U_{rat} \) was chosen, as similar subharmonic contamination was reported in a real power system [14]. For comparison, for Case A adjunct tests were performed for voltage subharmonics and interharmonics equal to \( U_{sh/ih} = 0.5\% U_{rat} \) (Fig. 7).

For \( U_{sh/ih}=1\% U_{rat} \), Case A (Fig. 4) and direction H, the vibration velocity rises from 1.34 mm/s for \( f_{sh}=5 \) Hz to 7.38 mm/s for \( f_{sh}=26 \) Hz and then falls to 0.571 mm/s for \( f_{sh}=45 \) Hz. For interharmonic injection, the vibration velocity is up to 1.82 mm/s. In direction V, the vibration velocity is much less and does not exceed 2.13 mm/s. For Case B (Fig. 5) and direction H, the vibration velocity is within the range of 5.27–6.95 mm/s for subharmonics frequency \( f_{sh} = 5–20 \) Hz. For subharmonics of greater frequency, the vibration velocity is up to...
Exemplary spectra of the voltage waveform are shown in Fig. 8 for this frequency. The interharmonic component is of frequency $f_{ih}=74$ Hz and value 0.42% $U_{rat}$, 0.1% $U_{rat}$ for Case A and Case B, respectively and the corresponding iTHD index (understood as the square root of the sum of squared per-cent values of subharmonics and interharmonics) – 1.19 % and 1.0 %. Presence of the voltage components results from the fact that an induction machine supplied with voltage containing subharmonics injects current interharmonics into a power system [20,27]. Their frequency is equal to $2f_i - f_{sh}$ (on the basis of [20]). The waveforms and spectra of the motor current for subharmonic of frequency $f_{sh}=26$ Hz are provided in Figs. 9–10 for Case A and Case B. For the former, the current subharmonics of value $I_{sh}=34.0\% I_{rat}$ (57.6\% $I_i$) is accompanied with interharmonic of value $I_{ih}=23.3 \% I_{rat}$ (39.6 \% $I_i$). For the latter, the current subharmonic is $I_{sh}=10.6 \% I_{rat}$ (18.1 \% $I_i$) and interharmonic – $I_{ih}=2.4 \% I_{rat}$ (4.1 \% $I_i$).

It is worth mentioning that the differences between both cases stem from the fact that high moment of load inertia (Case B) suppresses fluctuations of rotational speed and, as a result, reduces values of current subharmonics [31]. Consequently, the effect of voltage subharmonics on currents, power losses and windings temperature of an induction motor considerably depends on the moment of load inertia, and to a much lesser extent on load torque–speed characteristics [31]. For some frequencies, current subharmonics are much greater for small moments of load inertia than for large ones [31]. Also current interharmonics consumed by an induction motor under subharmonics are appreciably affected by the moment of load inertia.

Significant interharmonics level in the motor current results in voltage drop in the supply system. This could explain the presence of an interharmonic component in the supply voltage. A similar effect could case voltage drop along impedances of

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Significant interharmonics level in the motor current results in voltage drop in the supply system. This could explain the presence of an interharmonic component in the supply voltage. A similar effect could case voltage drop along impedances of
a real power system. It should be added that voltage subharmonics and interharmonics often occur together in a grid. Under voltage fluctuations, subharmonic of frequency \( f_{sh} \) is expected to be accompanied with an interharmonic of frequency \( 2f_i - f_{sh} \) (on the basis of [20]). For example, [17] reports simultaneous occurrence of voltage components of frequencies 8.1 Hz and 91.9 Hz. Moreover, interharmonics generally cause much less vibration level than subharmonics (see Figs. 4, 5, 6, 7).

To summarise, the presence of interharmonics in the testing voltage does not affect the conclusions resulting from this paper.

D. Torque Pulsations and Vibration Velocity

Under considered power quality disturbances in a machine are present two magnetic fields, excited by the fundamental voltage component and subharmonic or interharmonic frequencies [20]. As they rotate with various speeds, in electromagnetic torque and rotational velocity occur pulsations of frequency \( f_p \) equal: (based on [20])

\[
\begin{align*}
f_p &= f_i - f_{sh} \quad (11a) \\
f_p &= f_{sh} - f_i \quad (11b)
\end{align*}
\]

In practice, the pulsation frequency \( f_p \) is equal to the frequency of the main component of the vibration velocity under distorted voltage. Exemplary spectra of vibration velocity for Case A, frequency \( f_{sh} = 26 \text{ Hz} \), directions H, V, L are shown in Fig. 11, and for frequency \( f_{sh} = 70 \text{ Hz} \), direction H in Fig. 12. In Fig. 11, the first harmonic component of vibration is of frequency 23.75 Hz (and values 6.34 mm/s, 1.82 mm/s, 2.87 mm/s for directions H, V, L, respectively) which approximately corresponds to the pulsation frequency \( f_p \) described with (11a). The other frequency components are of much lower value. Similarly, in Fig. 12, the first harmonic component (20 Hz, 1.53 mm/s) corresponds to the frequency of torque pulsation, and the second one (25 Hz, 0.829 mm/s) to the rotational frequency.

It should be noted that for no load, the rotational torque alternately takes the positive and negative value. Possible interaction with the driven DC generator might justify high vibration for Case B. Further, for a fully loaded motor, the sign of the rotational torque will always be positive. What is more, in the fully loaded motor, the rotor is more stable because it is load down to the bearings. This could explain the comparatively low vibration level for Case C. Finally, as mentioned above, for Case A the current subharmonics are generally much greater than for Case B and Case C. As a result, they produce greater electromagnetic torques and forces than for the other cases, causing high vibration level. It should be noted that for Case A and frequency \( f_{sh} > 20 \text{ Hz} \), vibration velocity (Fig. 4) roughly corresponds to subharmonic content in the supply current (Fig. 13). Similarly to the vibration velocity, the current subharmonics take the largest value for frequency \( f_{sh} = 26 \text{ Hz} \).

It is also worth mentioning that pulsation of load torque (for example, for reciprocating compressors) may cause an increase in power losses as well as flow of current subharmonics and interharmonics, of frequencies \( f_i \pm f_p \) [44]. The negative phenomena [45] occur even under purely sinusoidal supply and are similar to those reported under subharmonics injection [27,31]. They are especially significant for the frequency of torsional excitation close to the natural one (eigenvalue) [45]. It should be noted that the frequency depends on machine dimensions. Generally, the greater the size of the machine, the lower the frequency. For
example, for an induction motor of rated power 37 kW, the natural frequency of the rigid-body mode is approximately 18 Hz, and for a 6 MW synchronous machine it is c. 3 Hz [44]. As voltage subharmonics also cause electromagnetic torque pulsations, resonance phenomena may explain the extraordinary vibration level and current subharmonic content for the frequency $f_{sh} = 26$ Hz. Under resonance of the rigid-body mode [45], electromagnetic torque due to current subharmonics and interharmonics boosts speed fluctuations, and speed fluctuations amplify current subharmonics and interharmonics. Speed fluctuations result in a reaction of the electromagnetic torque, which is compared to a torsional spring [45]. Currents, power losses and machine heating under voltage subharmonics and resonance of the rigid-body mode are analyzed in the authors’ paper [31]. It is worth adding that resonance for $f_{sh} = 26$ Hz corresponds to the natural frequency equal to 24 Hz.

A detailed explanation of the shape of the characteristics presented in Figs. 4, 5, 6, 7 will be a subject of future investigations. It is worth adding that vibration level is a function of motor speed and the subject is also for future investigations.

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**Fig. 11.** Spectra of vibration velocity for Case A, direction H (a), V (b), L (c) frequency $f_{sh} = 26$ Hz and $U_{sh} = 1\% U_{rat}$.

**Fig. 12.** Spectrum of vibration velocity for Case A, direction H frequency $f_{ih} = 70$ Hz and $U_{sh} = 1\% U_{rat}$.

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**Fig. 13.** Subharmonic and interharmonic content in the supply current vs. frequency of voltage subharmonics and interharmonics for Case A and $U_{sh}, U_{ih} = 1\% U_{rat}$. The frequency components are related to the rated current.

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**E. Discussion**

As mentioned above, the highest vibration level is observed for Case A and Case B. The broad-band vibration velocity vs. voltage subharmonic value $U_{sh}$ is shown in Fig. 14 for Case A and frequency $f_{sh} = 26$ Hz. The characteristic shows linear dependency for both direction H and V. For direction H, the vibration level reaches the boundaries of Zone D—4.5 mm/s—for $U_{sh} \approx 0.65\% U_{rat}$ and upper boundaries of Zone B—1.8 mm/s—for subharmonic voltage equal to merely $U_{sh} \approx 0.2\% U_{rat}$. In practice, in the case of the investigated machine for $U_{sh}$ greater than about 0.2 \% $U_{rat}$, the vibration level should be considered unacceptable for a long period of time. It should be stressed that the value $U_{sh} = 0.2\% U_{rat}$ is a few times lower than reported in real power systems [14,17].

To compound the problem, relatively minor subharmonic content can cause an excessive increase in windings temperature. For an exemplary 3-kW motor, the increase may exceed the recommended limits for voltage subharmonics greater than 0.4\% $U_{rat}$ [31].

The above examples illustrate the necessity of introducing the permissible levels of voltage subharmonics to power quality standards. Admittedly, the excessive vibration was observed only for no-load (Case A and Case B), but some motors work in idle conditions for the most of the operational time. An example is machines under duty type S6 15% [44]. It should be stressed that power quality standards are to be comprehensive and must provide effective protection of all
energy receivers against consequences of supplying with lowered voltage quality.

In summary, voltage subharmonics of values a few times lower than those occurring in real power systems may cause unacceptable vibration levels, thus requiring power quality standards to be revised.

VI. CONCLUSIONS

In power systems, voltage subharmonics and interharmonics of significant value may occur. They exert detrimental effects on synchronous generators, transformers, and energy receivers. Despite the extraordinary harmfulness of subharmonics and interharmonics, power quality standards do not impose limitations on them, as determination of their permissible levels requires “more experience” [32]. In previous works [21,23], there were certain attempts to indicate the maximum levels of subharmonics and interharmonics. The attempts were generally based on the effects of subharmonics on heating of induction motors and reduction of their start-up torque. The results of investigations presented in this paper reveal an additional detrimental effect of voltage subharmonics and interharmonics—namely, excessive vibration of induction motors. It was found that apparently even relatively minor traces of subharmonic contamination may cause undue vibration levels. For the investigated machine, even voltage subharmonics of only c. 0.2 % $U_{ref}$ can lead to unacceptable vibration. It should be stressed that the value 0.2 % $U_{ref}$ is a few times less than reported in real power systems [14,17]. Also, the presence of voltage interharmonics may result in excessive vibration. In the authors’ opinion, the need is urgent to introduce the permissible levels of voltage subharmonics and interharmonics to power quality standards. Determination of the appropriate limited values should be based on analysis of various undesirable phenomena caused by subharmonics and interharmonics, including vibration of induction motors.

VII. REFERENCES


39. Rotating electrical machines - Part 14: Mechanical vibration of certain machines with shaft heights 56 mm and higher - Measurement, evaluation and limits of vibration severity, IEC Standard 60034-14, 2003


